Why the Common Core changes math instruction

It's not the New Math exactly, but the Common Core calls for sharp changes in how math is taught and ultimately conceived in the earlier grades.

By Valerie N. Faulkner

At the heart of the Common Core standards is a move to create classroom discussions that clearly develop students' number sense by habitually making important connections across the mathematics (Hiebert & Stigler, 2004). Unfortunately, many of the habits students have learned and developed don't support these important mathematical connections. I created this guide to support teachers, administrators, and parents as they make important shifts in language to support implementation of the Common Core and discussion of sound mathematics.

VALERIE N. FAULKNER (Valerie_faulkner@ncsu.edu) is a teaching assistant professor in elementary education at North Carolina State University, Raleigh, N.C.

Teaching to the Common Core mathematics standards

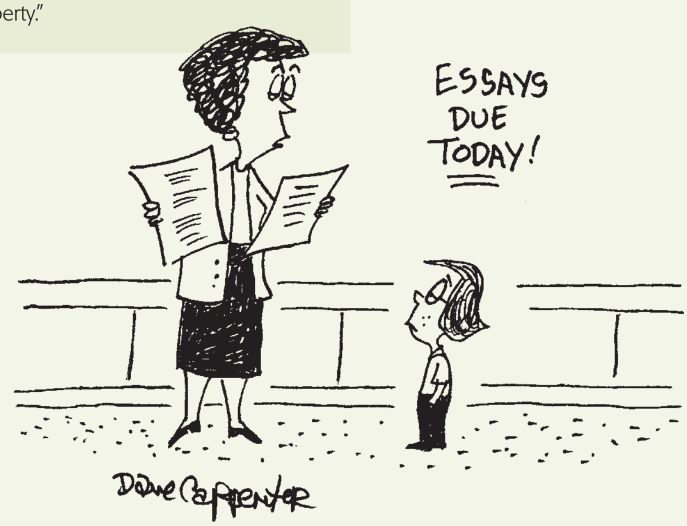
IMPORTANT NEW CHANGES

Eliminate this old habit	Adopt this new habit	Why?
Defining equality as "same as." For example: "Remember students, the equal sign means same as."	Define equality as "same value as." You might say this: "Remember students, the equals sign means same value as. The two values do not have to look alike, but they will have the same value. 3 + 4 tells a different math story than 4 + 3, but we know that they will both yield the same value of 7, so they are equal. Are they exactly the same? No, but they are equal."	The definition "same as" is mathematically incorrect and leads to misconceptions. Equals means that two things are the same based on one attribute — their quantitative value. Just as red or rough is an attribute, a thing's quantitative value also is an attribute. It is as absurd to say that 4 + 3 is the same as 1 + 6 as it is to say that a red truck is the same as a red lollipop. In the former, they have the same value; in the latter, they have the same color. In neither instance are they the same thing. Be precise (Carpenter et al., 1999; Carpenter, Franke, & Levi, 2003).
Calling digits "numbers." For example: "The numbers 7 and 3 are in 73."	Don't conflate digits with numerals. Instead clearly distinguish the difference between them, and take the time to be precise. You might say this: "Digits are in numerals. Numerals are written symbols that represent numbers. "73 has two digits — 7 and 3. 73 is a numeral that represents the number value of 73. 7 is a digit, and 3 is a digit within that numeral. In standard form, 73 is composed of the numbers 70 and 3."	It is important to distinguish between digits, numerals, and numbers. Numbers and numerals are essentially the same; numerals are the written form of the idea of the number. Digits and numerals are not the same. When we are imprecise about digits and numerals, we lose an opportunity to reinforce the workings of the base-10 number system. 7 is not a number when it is contained in 73. It does not mean 7 in the number sense. It means 7 in the digit sense and must be multiplied by a power of 10 to be fully understood (Ma, 2010).

Eliminate the old habit	Adopt this new habit	Why?
"Addition makes things get bigger."	Addition is about combining You might say this: "We use addition when we are combining two or more parts to make a total or when we are comparing pieces of information to figure out a total." OR: "When we add, we are taking a decomposed number and composing it into a simplified form."	Addition is about combining quantities, and it is only in elementary school where the numbers we combine are all positive numbers. By saying that addition makes things get bigger, we are: 1) Saying something that will have to be debugged in middle school. 2) Bypassing an opportunity to discuss the actual structures of addition. If I combine the $8 dollars I have with the $2 dollars I owe you, I will end up with $6. This is not more than my initial $8.
"Subtraction makes things get smaller."	Subtraction is about difference. You might say this: "We use subtraction when we're finding a missing part of a total or when we compare two numbers to find the difference between the two."	Subtraction does not make things get smaller. As above, this is a false construction based on a limited set of numbers that are introduced in elementary school. For instance: 5 minus -3 = 8. Consider in your own life the situation where a debt is taken away from you: "Don't worry about that $3 you owe me." Also consider the comparative model. If I compare what I have, let's say my net worth is $10,000, to what you have, let's say your net worth is a negative $5,000. When I compare those two numbers, the difference between our net worth is $15,000. This is actually the greatest of those three numbers!
"We don't have enough 1's so we need to go to the next place."	Support student understanding of how numbers are composed to support their understanding of the place value system. You might say this: "In this form, we don't have access to the 1's we need, so we need to change the form of the number." OR: "We don't have access to the 1's we need, and we need to get at those 1's." OR: "We have plenty of 1's. 25 is a larger number than 18 so we will even have some left over, but we need to decompose the higher unit value here so we can get at some of the 1's that are composed into 10's."	Research shows that elementary students don't understand that there are 10 1's in a 10. Language matters; students are literal. If they hear "we don't have enough 1's" whenever these problems are addressed, they begin to believe it. Shift your language to habitually point out that there are 1's available within the number, but that the form of the number is the issue. The 1's need to be accessed by decomposing the higher unit value (Chandler, C.C. & Kamii, C., 2009; Faulkner, 2009).
"You can't take a big number from a little number."	Prepare students for future learning rather than creating false notions about the number system. You might say this: "We could take a larger number from a smaller number, but we would get a negative number. You will learn about these later, but right now we will learn to solve this problem using all positive numbers."	This problem language is the cousin of the "we don't have enough 1's" mistake. It is critical that teachers do not make statements that are mathematically inaccurate in the service of a procedure or algorithm. We can teach efficient algorithms and maintain mathematically accurate language. Kids respond to this much better than we expect. At the very least, they hear accurate mathematical language from you, and, often this precision on your part leads to great conversations with the class. Here note the following: Traditionally I proceed as follows: 1 ~~2~~5 -18 7 But I am not really STUCK and forced to do this. I could use my knowledge of negative numbers as follows: 25 -18 -3 10 7 I CAN subtract 8 from 5. The result is -3. I can also subtract 10 from 20 and get 10. When I combine 10 and -3, I get 7. All subtraction problems of the type taught in 2nd grade could actually be solved by taking a larger number from a smaller number in this way.

Eliminate the old habit	Adopt this new habit	Why?
"Let's 'borrow' from the 10's place."	Compose the lower unit value/decompose the higher unit value. You might say this: "Here we will need to decompose a higher unit value to get the number into the form we want. "	Discontinue using the term "borrow." Use "regroup" and "trade" instead. Also try to include the concept of decomposing and composing the higher unit value. This language prepares students for situations beyond 10's and 1's, and for fractions and beyond. For example, decompose the higher unit value below to change the form of this mixed fraction to access thirds to solve this problem. $3\frac{1}{3}$ $-\ 2\frac{2}{3}$ In this form, I don't have access to the thirds I need. $3\frac{1}{3}$ is rewritten into the form of $2\frac{3}{3} + \frac{1}{3}$ or $2\frac{4}{3}$ so that I have access to the thirds that are composed into a whole. The problem then becomes: $2\frac{4}{3}$ $-\ 2\frac{2}{3}$ In this form, I can readily subtract. When I decompose the higher unit (ones) into the lower unit (thirds), I am acting on the same mathematical structure as with 10's and 1's. Our language should connect those ideas so that it does not appear to be a novel concept when taught in later grades (Ma, 2010).
Multiplication "makes things get bigger."	Teach the different structures of multiplication. There are three main structures for multiplication. One is that multiplication is repeated addition (measurement model); another answers the question of how many unique possibilities there are when matching one set with another (fundamental counting principle); the third finds a total amount or area when a column and row or two sides are known (area model). Try creating a story problem for each different structure type with the expression 4 x 3.	Multiplication only makes things get bigger in the limited world of positive whole numbers. As with addition and subtraction, focusing on the false idea that an operation does something to something else distracts from conversations about the structures of the given operation. Discussions about things 'getting bigger' may also distract students from the larger point of an equality. For instance 2 x 3 = 6. The biggest mathematical thought here is that two expression are equal to each other, not that something has 'gotten bigger.' Also, consider 4 x ½ or 4 x -1. In both cases, the product has a lesser value than the first factor (Ma, 2010).
Division "makes things get smaller."	Teach the different structures of division. There are three main structures for division. One structure involves repeated subtraction of groups (measurement model); the second answers the question of "how many for each one?" (partitive model/unit rate model); and the third model for division (area model) involves finding a side when an area and another side are known.	As with multiplication, division has just as much chance of making things smaller as it does of making things larger. In early grades, with only whole numbers to consider, it looks as if division behaves this way, or has that power. But this limited sample makes us think a behavior exists that does not exist. Your divisor, in combination with the act of dividing, determines the relative size of the quotient compared to the size of the dividend, not the operation of division itself. And again, remember that the overall equality between the expressions gets lost when we discuss the false idea that something has 'gotten smaller.' Consider: $6 \div \frac{1}{2} = 12$ OR $-6 \div -2 = 3$ In both cases here, the quotient is a larger number than the dividend and, in both cases, it is important to note that the expressions are equal.
"Doesn't go into." For example: "7 doesn't go into 3."	Prepare students for future learning You might say this: "We can divide 7 by 3, but the result won't be a whole number. When you begin working with fractions, you will solve problems like this regularly. Here we want to consider numbers that divide into other numbers without creating fractional parts or leftover pieces. "Consider if we had seven cookies and needed to split them between three people, what would happen? What if we had nine cookies between three people? In both cases, we can split up the cookies, but which one is easier? Why?"	Again, we need to make sure we maintain our accurate mathematical language even when something will always be true at our grade level. It isn't true that "7 doesn't go into 3." Even young children can understand the idea that in some cases there is a cookie left over that needs to be cut up in order for everyone to have equal shares. They know intuitively that I can have seven cookies and split them between three people. The language that says "you can't do that" separates their intuitive understanding with their academic mathematical understanding. Our job is to connect the intuitive with the academic.

Eliminate the old habit	Adopt this new habit	Why?
Saying "and" means decimal point.	Do not create false rules for language. When listening to and naming a number, consider the unit sizes that are being communicated. We know from language arts that "and" is a word that combines things. When people call numerals "and," it means to combine, but it also communicates a change in unit size. This is particularly important to think about when verbally communicating numbers with a decimal point in them. For example, one hundred and forty nine means literally 1 hundred unit and forty nine 1's units, while one hundred forty nine means one hundred forty nine 1's. Say what you mean. But if the form of the number does not matter to the case or argument, then it is fine to call a number in any form. In this same way, I could call 1,500 as fifteen hundred (15 hundreds) or one thousand five hundred (1,500 1's), or one thousand and five hundred (1 thousand and 5 hundreds). They imply different forms but they name the same number.	Saying that "and" means decimal point is an artificial construction. In common parlance and in math parlance, "and" generally means to combine, to add to, to augment. There is no reason to limit people's way of naming numbers or reading numerals. The key is that the numeral reader or number-namer comprehends the number they are communicating and does so in a fashion that allows the listener to comprehend the value as well. If I say one hundred and forty five, you know what I mean. I have effectively communicated the value 145 because "and" means, quite clearly, a combination of my 1 hundred and my 45 1's. When I say 100 and 45 and 37 one-hundredths, I effectively communicate the number I mean: 145.37. It may be a matter of taste to say that 145 and 37 hundredths is the most elegant way to say this number, but that is different than claiming that it is the only or best way to name it. To construct the idea that "and" means decimal point is inadvisable for two reasons: 1) It is not correct from a language perspective; and 2) It buries the opportunity to have a discussion that focuses on considering unit sizes and different ways to form a number. Whenever you get the chance to talk about units, place value meanings, and different forms of numbers, you want to do so!
Cancels out. For example: "These 8's cancel out."	Instead, explicitly use/discuss the property or idea that allows you to simplify. You might say this: "Here I have an 8 divided by an 8, and we know that anything divided by itself equals 1. So if I have 1 times something, what property can I use? Yes, the Multiplicative Identity Property. So you can see here that we have simplified this expression without disturbing/changing its value. "Here I have an X being subtracted from an X, and I know that anything minus itself equals 0. So, here I have 0 + something else. What property can I use here to make this expression simpler without changing its value? Yes, the Additive Identity Property."	When we bury these properties under the term "cancels out," students don't notice how often the properties are used and how important they are. Asking students to memorize the properties because they're important and then not pointing out the properties when they're used sends a mixed message to students that the properties are just facts rather than tools to be used regularly.



"No, 2's and 4's are not prepositions . . . "

PEDAGOGICAL CONSIDERATIONS

Eliminate the old habit	Adopt this new habit	Why?
Using "the answer." For example: What was the answer for the next question?	Using "the model," "the relationships," "the structure," or "justify your answer." You might say this: "Who can show how they modeled the next question? Did someone else do it a different way? "So what type of problem is this? Is it a part-part whole problem or a comparative problem? What in the story made you think that? Is it one you would solve using the structure of a linear equation? How do you know that? Is it a problem that involves a proportional relationship that you can use? What made you realize that? Is there another way to draw that relationship? "Justify your answer for us, and explain to us why you were able to switch those numbers in that way."	When our classroom habit is to find answers we forget to have the most important conversations of all: How did you do that? Why did you do that? What are the relationships that were important in this problem? How did you know that? What do mathematicians call it when you do that? Discussing the relationships in the problem rather than the "answer" helps to develop the important themes (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010).
Guess-and-check as a strategy.	Systematic mathematical representations, such as using bar models, are what teach students to think precisely as mathematicians think. Using base-10 to make a problem simpler is also a mathematical strategy.	While guess-and-check sometimes involves engaging number sense to locate an answer, this approach should not be taught as a strategy. Mathematical strategies involve instruction that prepares students for more difficult mathematics whereas guess-and-check is a habit that is not logical, or mathematical in its nature (Hoven & Garelick, 2007).

References

Carpenter, T.P., Fennema, E., Franke, M., Levi, L., & Empson, S. (1999). *Children's mathematics: Cognitively guided instruction*. Portsmouth, NH: Heinemann.

Carpenter, T.P., Franke, M., & Levi, L. (2003). *Thinking mathematically: Integrating arithmetic & algebra in elementary school.* Portsmouth, NH: Heinemann.

Chandler, C.C. & Kamii, C. (2009). Giving change when payment is made with a dime: The difficulty of tens and ones. *Journal for Research in Mathematics Education, 40* (2), 97-118.

Faulkner, V. (2009). The components of number sense: An instructional model for teachers. *Teaching Exceptional Children, 41* (5), 24-30.

Hiebert, J. & Stigler, J. (2004). Improving mathematics teaching. *Educational Leadership, 61* (5), 12-17.

Hoven, J. & Garelick, B. (2007). Singapore math: Simple or complex? *Educational Leadership, 65* (3), 28-31.

Ma, L. (2010). *Knowing and teaching elementary mathematics: Teachers' understanding of fundamental mathematics in China and the United States.* New York, NY: Routledge.

National Governors Association Center for Best Practices & Council of Chief State School Officers. (2010). *Common core state standards for mathematics.* Washington, DC: Authors.

"It's important to learn math because someday you might accidentally buy a phone without a calculator."